

ASPECT'24

Asymptotic Analysis and Spectral Theory 2024

Metz, September 23-September 27, 2024

Institut Élie Cartan de Lorraine,
Université de Lorraine, Metz, France














 *Access* 

France – Metz – Technopôle – 3 rue Augustin Fresnel – UFR MIM – Petit Amphithéâtre

By car Metz, 3 rue Augustin Fresnel
GPS : 49°05'41.5"N 6°13'47.2"E

By public transportation from either place de la république or Metz-Ville Train Station:
Bus: Mettis B – Direction: Hôpital Mercy – Stop: Grandes Écoles

Program

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
	09:00 – 09:45 C. Anghel	09:00 – 09:45 A. Hussein	09:30 – 10:15 S. Fliss	09:00 – 09:45 R. Yuncken
	09:50 – 10:35 M. Kohr	09:50 – 10:35 N. Raymond	10:15 – 10:40 C. Gomez Araya	09:50 – 10:35 J. Behrndt
				
	11:05 – 11:30 K. Naderi	11:05 – 11:30 L. Hermi	11:05 – 11:30 W. Spitzer	11:05 – 11:30 M. Rouveyrol
	11:30 – 11:55 B. Benhellal	11:30 – 11:55 E. Rosenberger	11:30 – 11:55 M. Zreik	11:30 – 11:55 S.K. Soni
	12:00 – 12:25 C. Dietze	12:00 – 12:25 S. Moroni	12:00 – 12:25 I. Karabash	
	12:25 – 12:50 M. Rouleux	12:25 – 12:50 D.T. Nguyen	12:25 – 12:50 S. Al Humaikani	
14:00 – 14:30 WELCOME				
14:30 – 15:15 K. Pankrashkin	14:30 – 15:15 S. Richard		14:30 – 15:15 V. Lotoreichik	
15:20 – 16:05 R. Bunoiu	15:20 – 16:05 V. Bruneau		15:20 – 16:05 N. Grosse	
				
16:30 – 16:55 S. Schraven	16:30 – 16:55 M. Nursultanov	16:30 – 18:30 GUIDED TOUR	16:30 – 16:55 C. Baudet	
16:55 – 17:20 L. Zielinski	16:55 – 17:20 G. Miranda		16:55 – 17:20 N. Weber	
		19:00 - 22:00 		

Social Events

Guided Tour of Metz Wednesday, 16:30–18:30

Meeting point at the Office du Tourisme (2 place d’Armes, 57000 Metz, near the Cathedral)

Conference diner Wednesday, 19:00, restaurant El Theatris, 2 place de la Comédie, 57000 Metz

CIPRIANA ANGHEL _____

Mathematisches Institut Göttingen

On the spectrum of the Dirac operator on degenerating Riemannian surfaces

We study the behavior of the spectrum of the Dirac operator on degenerating families of compact Riemannian surfaces, when the length of a simple closed geodesic shrinks to zero, under the hypothesis that the spin structure along the pinched geodesic is non-trivial. The difficulty of the problem stems from the non-compactness of the limit surface, which has finite area and two cusps. The main idea in this investigation is to construct an adapted pseudodifferential calculus, in the spirit of the celebrated b -algebra of Melrose, which includes both the family of Dirac operators on the family of compact surfaces and the Dirac operator on the limit non-compact surface, together with their resolvents.

JUSSI BEHRNDT _____

Technische Universität Graz

Weak coupling and spectral instability for Neumann and Krein Laplacians

We provide abstract criteria on spectral instability of nonnegative self-adjoint extensions of a symmetric operator and apply this to self-adjoint Neumann and Krein Laplacians on Lipschitz domains, intervals, and graphs. Our results can be viewed as variants of the classical weak coupling phenomenon for Schrödinger operators in $L^2(\mathbb{R}^n)$ for $n = 1, 2$.

This talk is based on joint work with Fritz Gesztesy (Baylor University, TX) and Henk de Snoo (University of Groningen).

VINCENT BRUNEAU _____

IMB, Université de Bordeaux

Spectrum of the Landau-Dirac operator perturbed by a variable sign potential

We consider the Dirac operator with constant magnetic field in the plan. Its spectrum consists of eigenvalues of infinite multiplicities: the Landau-Dirac levels. Under compactly supported perturbations of variable sign, near each Landau-Dirac level we study the distribution of the discrete eigenvalues. We explore some remarkable phenomena related to the finiteness or infiniteness of the discrete eigenvalues, which depend on the interplay of the different terms in the matrix perturbation.

This talk is based on a joint work with P. Miranda (Univ. de Santiago de Chile).

Homogenization of Maxwell's equations and related scalar problems with sign-changing coefficients

In this work, we are interested in the homogenization of Maxwell's equations posed in a composite medium with small, periodically distributed inclusions consisting of a negative material, namely a material modeled by a negative permittivity and permeability. By using T-coercivity techniques on the one hand and periodic homogenization on the other hand, we give sufficient conditions on the physical parameters which ensure the well-posed nature of the initial problem and the corresponding homogenized problem.

Based on joint work with L. Chesnel, K. Ramdani, M. Rihani

Edge states in rationally terminated honeycomb structures

We investigate the existence of edge states of a discrete "Laplace" operator corresponding to the tight binding model and defined in a sharp termination of a honeycomb periodic lattice along a straight line l (i.e. the intersection between the lattice and one of the half-spaces whose boundary is l). The answer is well known if the line runs in the so-called zigzag or armchair direction: In the first case there exists a non-dispersive edge states while in the second case there is no edge state. In this work, we consider an arbitrary line, but with rational directions: the truncated lattice is always periodic in the direction of this line, but the corresponding period can be arbitrarily large. We distinguish two classes of edges defined by the direction of the line, and we show that non-dispersive edge states exist for the first class, which we refer to as the zigzag type, while this is not true for the second class, which we refer to as the armchair type. Finally, we provide strong evidence that dispersive edge states can exist in both cases.

This is a joint work with Charles L. Fefferman (Princeton University) and Michael I. Weinstein (Columbia University).

On local boundary conditions for Dirac-type operators

We give an overview on smooth local boundary conditions for Dirac-type operators, giving existence and non-existence results for self-adjoint local smooth boundary conditions. We also discuss conditions when the boundary conditions are regular/Shapiro-Lopatinski (i.e. in particular giving rise to self-adjoint Dirac operators with domain in H^1) and talk about examples and obstructions in low dimensions.

This is joint work with Hanne van den Bosch (Universidad de Chile) and Alejandro Uribe (University of Michigan).

Non-self-adjoint scattering on graphs

Scattering theory for self-adjoint operators is a well-developed tool of mathematical physics. Here we consider a model case for non-self-adjoint operators: Laplacians on star graphs - that is on finitely many copies of the half line - subject to non-self-adjoint couplings at the origin. It is discussed for which couplings existence and completeness of the wave operators holds, in which cases this can be expected and in which not. The wave operators provide if existent and complete - as in the self-adjoint case - a representation of the absolutely continuous part of the operator.

On some elliptic operators on manifolds with cylindrical ends

We study some elliptic operators (Laplace, Stokes) on a manifold with cylindrical ends. To this purpose, we obtain useful Fredholm, regularity, and invertibility results. An important role is played by an adapted pseudodifferential calculus on manifolds with straight cylindrical ends which contains the inverses of its L^2 -invertible, elliptic operators of non-negative order. We also obtain the well-posedness of the corresponding Dirichlet problem.

Joint work with Victor Nistor (Metz) and Wolfgang L. Wendland (Stuttgart).

References

- [1] M. Kohr, V. Nistor, W.L. Wendland The Stokes operator on manifolds with cylindrical ends. *Journal of Differential Equations*, **407** (2024), 345D373.
- [2] M. Kohr, V. Nistor, W.L. Wendland Layer potentials and essentially translation invariant pseudodifferential operators on manifolds with cylindrical ends. ArXiv:2308.06308, (2023).

Magnetic Neumann Laplacian on a planar exterior domain

The central object of the talk is the magnetic Neumann Laplacian with homogeneous magnetic field in the complement of a bounded simply-connected smooth domain in the plane. The essential spectrum of this operator consists of the Landau levels, to which the discrete eigenvalues accumulate from below. We analyse the exterior of the disk in detail, observing that the ground-state eigenfunction is not radial and finding the accurate asymptotics of the low-lying eigenvalues in the weak field limit. We expect that the exterior of the disk is a global maximizer of the lowest magnetic Neumann eigenvalue within the class of complements of bounded simply-connected smooth domains of fixed area. In this talk, we will present a partial result in this direction, where the fixed-area constraint is substituted by a weaker constraint in terms of p-moments of the domain. Motivated by this eigenvalue optimization question, we also establish an asymptotic upper bound in the weak field limit on the lowest magnetic Neumann eigenvalue of the exterior of a bounded star-shaped domain.

This talk is based on a joint work with Ayman Kachmar and Mikael Sundqvist.

Eigenvalues of Laplacians with large negative Robin parameter: a review

The eigenvalues of Robin laplacians with large negative boundary parameter have attracted a lot of attention in the last 20 years due to interesting links between the eigenvalue asymptotics and the geometry of the boundary. We give an overview of available results for smooth and non-smooth domains and describe some open questions and ongoing works.

Spectral asymptotics of the Neumann Laplacian with variable magnetic field on a smooth bounded domain in three dimensions

This blackboard talk is devoted the semiclassical spectral analysis of the Neumann magnetic Laplacian on a smooth bounded domain in three dimensions. Under a generic assumption on the variable magnetic field (involving a localization of the eigenfunctions near the boundary), we establish a semiclassical expansion of the lowest eigenvalues. Thereby, we solve an open question from the Ph.D. thesis of the speaker.

Joint work with Maha Aafarani, Khaled Abou Alfa and Frédéric Hérau.

The topological nature of resonance(s) for 2D Schroedinger operators

In 1986, Gesztesy et al. revealed the surprising behavior of thresholds resonances for two-dimensional scattering systems: their contributions to Levinson's theorem are either 0 or 1, but not $1/2$ as previously known for systems in dimension 1 and 3. During this seminar, we shall review this result, and explain how a C^* -algebraic framework leads to a better understanding of this surprise. The main algebraic tool consists of a hexagonal algebra of Cordes, replacing a square algebra sufficient for systems in 1D and 3D. No prior C^* -knowledge is expected from the audience.

This presentation is based on a joint work with A. Alexander, T.D. Nguyen, and A. Rennie.

The Helffer-Nourrigat conjecture : hypoellipticity of polynomials of vector fields

A differential operator P is hypoelliptic if Pf smooth implies f smooth. Two famous theorems give large classes of hypoelliptic operators: Hormander's sums-of-squares theorem and the Rockland Conjecture, proven by Helffer & Nourrigat. These two examples were unified in a conjecture by Helffer-Nourrigat from 1979. I will describe how the Helffer-Nourrigat conjecture can be proven using the groupoid approach to pseudodifferential operators.

Joint work with Androulidakis & Mohsen.

Sarah Al Humaikani

POEMS (CNRS, INRIA, ENSTA Paris)

A Rellich-type theorem for the Helmholtz equation in a junction of stratified media

Some years ago, the following result was proven: there are no non-trivial square-integrable solutions to the Helmholtz equation in a two-dimensional unbounded conical domain with opening angle larger than π . We prove that this result can be generalized to some Helmholtz equations with non-constant coefficients. More precisely, the conical domain must be replaced by a union of half-planes, such that each half-plane is either homogeneous or stratified with a stratification orthogonal to the boundary of the half-plane. Our proof is based on half-plane representations of the solution derived through a generalization of the Fourier transform adapted to stratified media. In particular, this result proves the absence of trapped modes at the junction of stratified media as soon as the angles between branches are greater than $\pi/2$.



Cédric Baudet

POEMS, CNRS, Inria, ENSTA Paris

Asymptotic analysis at any order of Helmholtz's problem in a corner with a thin layer

We consider Helmholtz's equation in an angular sector covered on one side by a layer, where the sector and the layer are two homogeneous media. We build an asymptotic expansion at any order of the solution when the thickness of the layer tends to 0. This is done using matched asymptotic expansion, which consists here in introducing different asymptotic expansions of the solution in three subdomains: the vicinity of the corner, the layer and the rest of the domain. These expansions are linked through matching conditions. The presence of the corner makes these matching conditions delicate to derive because the fields have singular behaviors. Our approach is to reformulate these matching conditions purely algebraically by writing all asymptotic expansions as formal series. By using algebraic calculus we reduce the matching conditions to scalar relations linking the singular behaviors of the fields. These relations have a convolutive structure and involve some

coefficients that can be computed analytically. Our asymptotic expansion is justified rigorously with error estimates.



Badreddine Benhellal

University of Oldenburg

Spectral asymptotic for Schrödinger operators with oblique transmission condition on Lipschitz curves

I will address the self-adjointness and spectral properties of a two-dimensional Schrödinger operator with an oblique-type transmission condition along a compact Lipschitz curve Σ involving the normal and tangential derivatives and a coupling parameter α . After showing the self-adjointness and the basic spectral properties, I will discuss some spectral asymptotics for the case $\alpha < 0$, where the discrete spectrum turns out to be unbounded from below. I will provide a first-order asymptotic expansion of the eigenvalues of the underlying operator as $\alpha \rightarrow 0^-$ under a certain assumption on the asymptotic expansion of the eigenvalues of the Schrödinger operator with a large attractive delta interaction on Σ . I will also give an example where our assumption is satisfied and the main term of the asymptotic is different from the case of smooth curves.



Charlotte Dietze

LMU Munich

Weyl formulae for some singular metrics with application to acoustic modes in gas giants

This paper is motivated by recent works on inverse problems for acoustic wave propagation in the interior of gas giant planets. In such planets, the speed of sound is isotropic and tends to zero at the surface. Geometrically, this corresponds to a Riemannian manifold with boundary whose metric blows up near the boundary. Here, the spectral analysis of the corresponding Laplace-Beltrami operator is presented and the Weyl law is derived. The involved exponents depend on the Hausdorff dimension which, in the supercritical case, is larger than the topological dimension. This is joint work with Yves Colin de Verdière, Maarten de Hoop and Emmanuel Trélat.



Camilo Gomez Araya

CEREMADE, Paris Dauphine

Edge states for tight-binding operators with soft walls

This talk concerns with tight-binding models that describe periodic structures terminated by a soft wall. Our main examples are the one-dimensional SSH chain and the two-dimensional Wallace model for graphene. The soft wall is just a potential that increases smoothly to infinity on one side, thus forbidding propagation to that side. We prove that a spectral flow appears in these corresponding edge models, as the wall is shifted. We identify this flow as a number of Bloch bands, and provide a lower bound for the number of edge states appearing in such models. Joint work with David Gontier and Hanne Van Den Bosch.



Lotfi Hermi

Florida International University, Miami

On a nonlocal integral operator commuting with the Laplacian and the Sturm-Liouville problem: Low rank perturbations of the operator

We reformulate all general real coupled self-adjoint boundary value problems as integral operators and show that they are all finite rank perturbations of the free space Green's function on the real line. This free space Green's function corresponds to the nonlocal boundary value problem proposed earlier by Saito [Appl. Comput. Harmon. Anal. 25, 68–97 (2008)]. We prove these perturbations to be polynomials of rank up to 4. They encapsulate in a fundamental way the corresponding boundary conditions. (This is joint work with Naoki Saito, University of California, Davis)



Illia Karabash

University of Bonn

Boundary tuples, m-dissipative boundary conditions, and discrete spectra of Maxwell operators.

Motivated by the modelling of open optical cavities, we introduce m-boundary tuples as a generalization of boundary triples, and use them for

a description of all m-dissipative boundary conditions for 3-d Maxwell systems in Lipschitz domains. This result is applied then to the class of generalized impedance boundary conditions. We plan to discuss also additional assumptions that guarantee the discreteness of spectra associated with generalized impedance boundary conditions. The talk is based on two joint papers with Matthias Eller, doi.org/10.1016/j.jde.2022.04.006 and doi.org/10.48550/arXiv.2401.01049.



German Miranda

Lund University

The Magnetic Laplacian on the disc for strong constant magnetic fields.

The study of the Neumann realization of the magnetic Laplacian traces back to works of Saint-James and de Gennes in the context of superconductivity. On a planar domain and under a strong magnetic field, the magnetic Laplacian has eigenvalues close to the Landau levels. When the domain is a disc, the spectrum consists of branches of eigenvalues of one dimensional operators. Using a variational approach, we derive asymptotics of the eigenvalues of the Neumann realization of the magnetic Laplacian under a strong constant magnetic field with accurate estimates of exponentially small remainders. Our approach allows us to recover recent results by Baur and Weidl for the Dirichlet realization. This is joint work with Ayman Kachmar.



Sergio Moroni

BCAM, Bilbao

Stability of Optical Solitons in 2d

The optical properties of nematic liquid crystals have garnered significant attention in recent years due to their ability to support stationary optical waves. Owing to their high susceptibility, the response of a nematic liquid crystal to a light beam passing through it is both nonlocal and nonlinear. This response induces a self-focusing effect on the light beam, which creates waveguides that counteract the natural diffraction spreading of the beam, and, when optimally shaped, allows for the existence of stationary waves.

We study the ground states of the Schrödinger-Poisson system in dimension $(2 + 1)$

$$i\partial_z u + \frac{1}{2}\Delta u + u \sin(2\theta) = 0$$

$$-v\Delta\theta + q \sin(2\theta) = 2|u|^2 \cos(2\theta)$$

that is proposed as a model for the propagation of a laser beam through a planar cell filled with a nematic liquid crystal. We present a first stability result for those stationary waves. This provides a strong justification of the relevance of the mathematical model to applications, as only locally stable solutions are expected to be seen in experiments and numerical simulations.

Kiyan Naderi

Carl von Ossietzky Universität Oldenburg

An embedded trace theorem for infinite metric trees with applications to transmission problems with mixed dimensions

For a class of weighted infinite metric trees we propose a definition of the boundary trace which maps H^1 -functions on the tree to L^2 -functions on a compact Riemannian manifold. For a range of parameters, the precise Sobolev regularity of the traces is determined. This allows one to show the well-posedness for a Laplace-type equation on infinite trees interacting with Euclidean domains through the boundary. Based on joint works with Valentina Franceschi (Padova), Maryna Kachanovska (Paris) and Konstantin Pankrashkin (Oldenburg).

Din Thi Nguyen

Uppsala University

2D focusing "almost-bosonic" anyons gases.

In the two-dimensional space, we consider a trapped system of N anyons interacts via an attractive two-body interaction. The maximum value of the interaction strength is defined by the "magnetic" Gagliardo-Nirenberg inequality. In the stable regime, we derive the average-field-Pauli functional (also known as Chern-Simon-Schrodinger) as the mean-field limit of many-body quantum mechanics.

Medet Nursultanov

University of Helsinki

Spectral Theory for 1D Schrödinger operators with measure potentials

We investigate the spectral properties of 1D Schrodinger operators with measure potentials. We obtain two-sided estimates for the spectral distribution function of the eigenvalues. As a corollary, we derive a criterion for the discreteness of the spectrum and a criterion for the membership of the resolvents to Schatten classes. We give two side estimates for the lower bound of the essential spectrum.

Elke Rosenberger

Universität Potsdam

Weyl asymptotics for discrete pseudo-differential operators

For a class of elliptic self-adjoint pseudo-differential operators, we give asymptotic estimates for the number of eigenvalues in a fixed spektral interval. Here we assume the related symbols to be periodic with respect to momentum. The associated operators acts on functions on a lattice scaled by a semi-classical parameter. If the boundaries of the spectral Intervall are non-critical values of the principal symbol, we show that the exact leading order term for the number of eigenvalues is given by the phase space volume of the pre-image of the interval under the principal symbol.

Michel Rouleux

Université de Toulon

Generalized exchange operators for a system of spin-1 particles, and application to the spectrum of interaction Hamiltonians

The irreducible representations (reps) $(SU(2), \mathcal{H}, U)$ of $SU(2)$ of dimension $(2S + 1)^N$, i.e. operators acting on the space $\mathcal{H} = \mathbf{C}^{(2S+1)^N}$ of N identical particles with spin S , are described by Clebsch-Gordan decomposition into inequivalent irreducible reps.

In the special case $S = 1/2$, Dirac (1929) discovered that there is an equivalent representation given by $(\mathcal{S}(N), \mathcal{H}, V)$ where $\mathcal{S}(N)$ is the permutation group. Thus, the standard "linear" Hamiltonian, or Heisenberg interaction Hamiltonian $H_0 = \sum_{1 \leq i < j \leq N} \vec{\sigma}_i \cdot \vec{\sigma}_j$, where $\vec{\sigma}_i = 2\vec{S}_i$ is the vector of Pauli

matrices, can be interpreted as the sum of the “Exchange Operators” P_{ij} exchanging particle i with particle j . Schrödinger (1941) constructed the Exchange Operator $P_{ij} = P_S(\vec{S}_i \cdot \vec{S}_j)$ as a polynomial of degree $2S$ in $\vec{S}_i \cdot \vec{S}_j$. This we call the P -representation. There is another rep induced by the one particle permutation of states operators \tilde{Q}_α , which we call the Q -rep. P - and Q -reps are not equivalent. We are mostly interested in the simplest case where there are so many particles as available states for the spin operator along the z -axis, i.e. $N = 2S + 1 = 3$, see Weyl (1946) or Hamermesh (1962).

We then address the problem of constructing a commutative Lie algebra L_1 with generators $J_1 = \sum_\alpha Q_\alpha$, $J_2 = \sum_{\alpha < \beta} [Q_\alpha, Q_\beta]$ (up to a correction) and $J_3 = \sum_{\alpha < \beta < \gamma} [Q_\alpha, [Q_\beta, Q_\gamma]]$, together with its dual algebra L_1^* , so that $L_1 \oplus L_1^*$ contains Heisenberg (spin 1) interaction Hamiltonian H_0 . Here \tilde{Q}_α is the lift of Q_α to the total space. We expect that other rotation invariant Hamiltonians (such as Schrödinger Hamiltonian H_S) also belong to the enveloping algebra of $L_1 \oplus L_1^*$, and that their spectrum and eigenvectors are still integer valued (up to trivial factors), which reflects the fact that all irreps of $\mathcal{S}(N)$ are rational.

Marc Rouveyrol

Univ. Paris-Saclay (Orsay)

Propagation of smallness and spectral estimates on non-compact manifolds

Spectral estimates consist in bounding the norm of frequency-localized functions by their norm on a smaller, so-called “sensor” set, up to some factor depending on the frequency threshold. They were first studied by Logvinenko and Sereda in the 1970s for flat Laplacians, in connection with the uncertainty principle. Recent investigations have focused on the manifold case, with tools drawing from spectral theory, harmonic analysis, geometric analysis and control theory for the heat equation. Beyond controllability results, applications of these estimates include spectral geometry and the study of random Schrödinger operators.

The aim of the talk will be to give an introduction to spectral estimates and their link with controllability, as well as to present the first high-frequency results on non-compact, non-flat Riemannian manifolds. In the model case of the hyperbolic half-plane,

we will give an optimal equidistribution condition on the sensor set, equivalent to spectral estimates. If time allows, we will discuss generalizations to hyperbolic surfaces, on which decay in the volume of balls creates additional difficulties. Based on joint work with Alix Deleporte.

Severin Schraven

Technical University of Munich

Two-sided Lieb-Thirring bounds

We discuss upper and lower bounds for the number of eigenvalues of semi-bounded Schrödinger operators in all spatial dimensions. For atomic Hamiltonians with Kato potentials one can strengthen the result to obtain two-sided estimates for the sum of the negative eigenvalues. Instead of being in terms of the potential itself, as in the usual Lieb-Thirring result, the bounds are in terms of the landscape function, also known as the torsion function, which is a solution of $(-\Delta + V + M)uM = 1$ in \mathbb{R}^d ; here $M \in \mathbb{R}$ is chosen so that the operator is positive. This talk is based on the preprint arXiv:2403.19023 which is joint work with S. Bachmann and R. Froese

Sandeep Kumar Soni

University of Zagreb, Croatia

Non-stationary theory for Friedrichs systems

The theory of abstract Friedrichs operators, introduced by Ern, Guermond and Caplain (2007), proved to be a successful setting for studying positive symmetric systems of first order partial differential equations (Friedrichs, 1958), nowadays better known as Friedrichs systems. Recently, a characterisation of abstract Friedrichs operators in terms of skew-symmetric operators and bounded selfadjoint operators has been established. In this presentation we shall see the non-stationary theory of abstract Friedrichs operators along with the theory of skew-symmetric operators. We use the von Neumann extension theory for the connection between the theories of these two types of operators. A boundary quadruple/triplet approach has been used to study the semigroup theory.

Wolfgang Spitzer

FernUniversitaet in Hagen

Entanglement entropy in the ground state of non-interacting massless Dirac fermions in dimension one

We present a novel proof of a formula of Casini and Huerta for the entanglement entropy of the ground state of non-interacting massless Dirac fermions in dimension one localized to (a union of) intervals and generalize it to the case of Rényi entropies. At first, we prove that these entropies are well-defined for non-intersecting intervals. This is accomplished by an inequality of Alexander V. Sobolev. Then we compute this entropy using a trace formula for Wiener–Hopf operators by Harold Widom. For intersecting intervals, we discuss an extended entropy formula of Casini and Huerta and support this with a proof for polynomial test functions (instead of entropy).



Nicolas Weber

Technische Universität Graz

Weakly coupled Schrödinger operators with complex potentials

We consider the (not necessarily self-adjoint) Schrödinger operator of the form $H_\beta = -\Delta - \beta V$ in $L^2(\mathbb{R}^d)$, $d \in 1, 2$, where $V : \mathbb{R}^d \rightarrow \mathbb{R}$ is a real-valued decaying potential, and $\beta \in \mathbb{C}$ is a small complex-valued parameter. We derive sufficient conditions on the real part $\text{Re}(\beta)$ of β , depending on the spatial dimension and V , for the existence of an eigenvalue $\lambda_\beta \in \mathbb{C} \setminus [0, \infty)$ of H_β as $\beta \rightarrow 0$, and also obtain an asymptotic expansion for this eigenvalue. Finally, we provide conditions on $\text{Re}(\beta)$, under which H_β has no eigenvalues in $\mathbb{C} \setminus [0, \infty)$ as $\beta \rightarrow 0$.



Lech Zielinski

ULCO, PRES Université Lille-Nord de France

On the second order approximation of large eigenvalues for the quantum Rabi model

The QRM (Quantum Rabi Model) describes a two level quantum system interacting with a single mode radiation. It plays a fundamental role in quantum optics, quantum information and condensed matter physics. The Hamiltonian of the QRM has discrete

spectrum and depends on two parameters: the spacing between the levels of the quantum system and the coupling constant. A lot of research works has focused on the approximation of its eigenvalues. The most popular approach is based on the first order perturbation theory and bears the name of the GRWA (generalized rotating-wave approximation, see [1]). It appears that the three-term asymptotic formula for large eigenvalues of the QRM proved in [2] is closely related with the GRWA and allows one to determine the absolute value of parameters from the spectrum in [3]. The purpose of this talk is to present the three-term asymptotic formula for large eigenvalues of the QRM with a static bias, which is a basic model of the experimental circuit quantum dynamics. The additional difficulty is due to the fact that the general case requires the use of the second order perturbation theory. The talk is based on joint work with R. Alsayed, A. Boutet de Monvel and M. Charif.



Mahdi Zreik

Université de Toulouse 3 - Paul Sabatier

On the Self-adjointness of two-dimensional relativistic shell interactions

In this talk, I will discuss the self-adjointness of the two-dimensional Dirac operator coupled with a singular combination of electrostatic and Lorentz scalar δ -interaction, supported on a closed Lipschitz curve. The main new ingredients are an explicit use of the Cauchy transform on non-smooth curves and a direct link with the Fredholmness of a singular boundary integral operator. This results in a proof of self-adjointness for a new range of coupling constants, which includes and extends all previous results for this class of problems. The study is particularly precise for the case of curvilinear polygons, as the angles can be taken into account in an explicit way. In particular, if the curve is a curvilinear polygon with obtuse angles, then there is a unique self-adjoint realization with 1 domain contained in H^2 for the full range of non-critical coefficients in the transmission condition.

The results are based on a joint work with Badreddine Benhellal and Konstantin Pankrashkin.

Participants

Sarah AL HUMAIKANI — POEMS (CNRS, INRIA, ENSTA Paris)
Cipriana ANGHEL STAN — Mathematisches Institut Göttingen
Cédric BAUDET — POEMS, CNRS, Inria, ENSTA Paris
Badreddine BENEHELLAL — University of Oldenburg
Jussi BEHRNDT — Technische Universität Graz
Vincent BRUNEAU — IMB, Université de Bordeaux
Renata BUNOIU — University of Lorraine, Metz
Charlotte DIETZE — LMU Munich
Alessandro DUCA — INRIA Nancy
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Nadine GROSSE — University of Freiburg
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Camilo GÓMEZ ARAYA — Université Paris Dauphine
Stefan HALLER — University of Vienna
Lukáš HERIBAN — Czech Technical University in Prague
Lotfi HERMI — Florida International University
Amru HUSSEIN — RPTU Kaiserslautern-Landau
Illia KARABASH — University of Bonn
Mirela KOHR — Babes-Bolyai University
Massimo LANZA DE CRISTOFORIS — Università degli Studi di Padova
Julien LEQUEURRE — University of Lorraine
Vladimir LOTOREICHIK — Czech Academy of Sciences
Marwa MANSOURI — ESSTH Sousse
Germàn MIRANDA — Lund University
Sergio MORONI — BCAM Bilbao
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Sandeep Kumar SONI — University of Zagreb
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Eric VACELET — Université d’Angers
Nicolas WEBER — Graz University of Technology
Robert YUNCKEN — University of Lorraine
Lech ZIELINSKI — ULCO, PRES Université Lille-Nord de France
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